

An Introduction to Derivatives and Risk Management, 10th Edition

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Technical Note: Margin Trading and Short Selling

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In investments courses, the concepts of margin trading and short selling are fairly well covered. But in a course in derivatives or corporate finance, they are often treated only lightly, if at all. They are very important, especially in a derivatives course where you will frequently come into contact with the notions of taking a short position via a derivative or taking a long position via a derivative with effectively a significant amount of leverage through low margin requirements. Thus, it is important to first understand how margin trading and short selling work in the securities markets. These concepts are not very intuitive, so you should find this note helpful.

Basic Concept of Return

Most people understand the notion of buying a stock. When you purchase a stock you pay the current price, say S_0 . Let us first assume there are no dividends over your holding period. At the end of the holding period, let the price be S_T . Then your rate of return is the percentage price change:

$$r_t = \frac{S_T - S_0}{S_0},$$

or

$$r_t = \frac{S_T}{S_0} - 1.$$

Typically you would purchase a stock with the hope that its price would increase, thereby leading to a positive rate of return.

Returns with Dividends

Let us say that at the end of your holding period, the stock pays a dividend of D_T . Then you will not only have a claim on the stock's value, S_T , but you will also receive the dividend D_T .¹ Thus, the return would be

$$r_t = \frac{S_T + D_T}{S_0} - 1.$$

Margin Trading

Now let us consider margin trading. The notion of buying a stock (or bond) on margin is to borrow some of the purchase price and pay it back later with interest. The borrowed money results in a degree of leverage, as we shall see below, that amplifies gains if the stock does well at a cost of amplifying losses if the stock does poorly.

Although the term “margin” is sometimes erroneously thought to be the amount borrowed, it is actually the amount not borrowed, which should be thought of as the investor's equity or the amount of the investor's personal funds in the investment. For example, suppose a stock costs \$100 and you decide to borrow a portion of its price, thereby putting up your own money in the amount of the difference between \$100 and the amount you borrow. Suppose we consider two possible prices to which the stock could go: it could fall 50% to \$50 or rise 50% to \$200. In the table below, we show the rate of return under the two cases of 100% margin and 60% margin. To keep things as simple as possible and to focus on the issue that matters the most, we assume no dividends. Assume that the investment horizon is one year and that the margin loan has an interest rate of 5%.

Arrangement	Margin	Return if stock goes to	
		\$50	\$200
Borrow \$0, invest \$100	\$100 or 100%	$\frac{\$50}{\$100} - 1 = -50\%$	$\frac{\$200}{\$100} - 1 = 100\%$
Borrow \$40, invest \$60	\$60 or 60%	$\frac{\$50 - \$40(1.05)}{\$60} - 1 = -86.67\%$	$\frac{\$200 - \$40(1.05)}{\$60} - 1 = 163.33\%$

¹ If the dividend is paid during the holding period, the value D_T should reflect the dividend plus any interest you could earn on it during the remainder of the holding period.

Notice that when the stock does well, the return is magnified in an upward direction. When the stock does poorly, the return is magnified in a downward direction.

In studying investments, we begin by looking at how an investor allocates his money among two assets. Suppose one asset is a risky asset and one is risk-free. Let $E(r)$ be the expected return on the risky asset, r_F be the expected return on the risk-free asset, which is the so-called risk-free rate. The risky asset has a standard deviation of σ . Let w_R be the percentage of funds allocated to the risky asset and w_F be the percentage of funds allocated to the risk-free asset. These percentages must add up to 1, i.e., $w_R + w_F = 1$, because all funds must be accounted for.

We know that the expected return on the overall portfolio, $E(r_p)$, is a weighted average of the expected returns on the risky asset and risk-free asset.

$$E(R_p) = w_R E(r) + w_F r_F.$$

The variance of a two-asset portfolio is a weighted-average of the squared weights of each asset times the variance of that asset plus a term that reflects the covariance. But since the risk-free asset has no variance, it has no covariance so the portfolio variance and standard deviation are simplified to

$$\begin{aligned}\sigma_p^2 &= w_R^2 \sigma_R^2 \\ \sigma_p &= w_R \sigma_R.\end{aligned}$$

If the investor has a portion of his money invested in the risky asset and a portion invested in the risk-free asset, then $0 < w_R < 1$ and $0 < w_F < 1$. If an investor trades on margin, he borrows some money by issuing the risk-free asset as a loan. Thus, we assume he borrows at the risk-free rate. The amount borrowed divided by the total amount of money the investor's money is the value w_F , subject to this number being negative.² Thus, if the investor has \$100 and borrows \$40, then $w_F = -\$40/\$100 = -0.40$. If the investor has \$100 and borrows \$40, he now has \$140 with which to invest in the risky asset. The weight assigned to the risky asset, w_R , is the amount invested divided by the investor's wealth. In this case, it is $\$140/\$100 = 1.40$. Again, note that the weights sum to

²Think of the amount borrowed as a loan. If you borrow some money, it is a liability, which is usually represented by a negative value. Remember, equity = assets – liabilities.

1. The above formulas for expected return and variance apply regardless of the signs of the weights. If margin trading is done, then w_R is larger than 1 and w_F is smaller than 0. Given that $E(r) > r_F$, margin trading will increase the portfolio expected return.³ Note, however, that portfolio risk will also increase. The variance formula is increasing in w_R . Thus, if you borrow to invest more in the risky asset, the overall risk increases. That is indeed what we saw in the above table.

Short Selling

Now let us take a look at short selling. The securities markets offer the opportunity for an investor to borrow a stock or bond. The idea is that you would borrow the security, sell it, and then hopefully watch the stock price fall. Then you would buy it back later at what would hopefully be a lower price. You are selling the stock at a given price and buying it back later at a lower price, thereby earning a return but in the manner of selling first and buying later instead of the conventional manner of buying first and selling later.

The return on a short sale is calculated the same way as on a long position (see the formula at the beginning of this note) but does require a slightly different interpretation. Consider this example. Suppose you borrow a stock priced at \$100 and sell it. The stock price falls to \$95 and you buy it back. What is your return? According to the above formula, it is

$$r_i = \frac{\$95}{\$100} - 1 = -5\%.$$

Most people would argue that this seems wrong. You sold something for \$100 and bought it for \$95. How can it be that you lost 5%? To resolve this puzzle, you must understand that a short sale should be interpreted like a loan. It is like you borrowed \$100 and had to pay back only \$95. Now if you had done a standard loan, borrowing \$100 at 4%, you would pay back \$104, and you would say this is just a loan with a 4% interest rate. In a short sale, however, you can have a negative effective rate on the loan. In our example, you borrowed \$100 and paid back \$95. That is an effective rate of -5%. Under

³ $E(r) > r_F$ because logically riskier investments should earn more than risk-free investments.

this interpretation, you should agree that anyone would be happy to borrow at a negative interest rate. You end up paying back less than you borrowed.

With short sales, you want your rate of return to be as low as possible. There is a theoretical limit, however, of -100% . The stock can go no lower than zero, so it is remotely possible you would borrow \$100 and have the stock go to \$0, thereby resulting in a loan with a rate of -100% .

Before thinking that this is the perfect type of loan, we must also examine the risk. It is quite large indeed. In fact, the upside risk is infinite. Because there is no upper limit on a stock price, the rate of return can be infinite and this is certainly not a desirable outcome to a short seller. That is, there is the possibility that you will have to pay back an infinite amount of money!

In modern portfolio theory, there is no basis on which to justify short selling a risky asset to purchase more of the risk-free asset.⁴ But there is a basis to justify short-selling one risky asset to purchase another. Let us create two assets, called Asset 1 and Asset 2, with expected returns of $E(r_1)$ and $E(r_2)$, standard deviations of σ_1 and σ_2 , and a correlation of ρ_{12} . We know that the portfolio expected return and variance are the formulas

$$E(r_p) = w_1E(r_1) + w_2E(r_2)$$
$$\sigma^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2.$$

Let stock 1 be selling for \$100 and stock 2 be selling for \$20. Suppose our portfolio consists of one share of stock 1. That is, we have \$100 and it is invested entirely in one share of stock 1. Suppose we sell short 10 shares of stock 2 and use the proceeds to buy additional shares of stock 1. Selling short 10 shares of stock 2 would mean borrowing 10 shares of stock 2, selling those shares, and generating \$200 in cash. We could then buy two more shares of stock 1. Now we would have three shares of stock 1 worth \$100 each for a total of \$300 and be short 10 shares of stock 2 worth \$200. Our personal balance sheet would reflect \$300 of stock owned as assets and \$200 of stock sold short as liabilities for a net worth of \$100, which indeed is the amount of money we

⁴In other words, one would not take on additional risk in order to earn more of a risk-free return.

said we started with. The weights are $w_1 = \$300/\$100 = 3.0$, and $w_2 = -\$200/\$100 = -2.0$ and they clearly sum to 1.

The above formulas for the portfolio expected return and variance are correct. We just have to make sure we use the right weights. Nonetheless, short selling does not necessarily increase the risk. In fact, it can decrease it. Logically, going long one stock and short another can clearly decrease the risk if the two stocks move in the same direction. Of course, there is more to it than this, but this is what you study in modern portfolio theory.

Dividends on Margin Trading and Short Selling

If there are dividends on the stock, margin investors receive the full dividends. They are simply using their borrowed funds to purchase more shares, so they get the dividends on the additional shares. Dividends create a somewhat confusing result when short selling. Short sellers have to pay dividends. Recall in the example above, that we had \$100 all invested in stock 1 and we borrowed 10 shares of stock 2 priced at \$20. If there are any dividends on stock 1, the short seller certainly gets these dividends. But if there are any dividends on stock 2, the short seller must pay the dividends to the party from who the shares were borrowed.

Let the short seller be an investor, call him Short, and the party lending the shares be a broker who we shall call Broker. The broker must obtain the shares somewhere. He can lend them from his own or his firm's portfolio, he can go out and buy them, or he can borrow them from one of his customers. This third method is quite common and the party owning the shares may never know that his shares have been lent. But that is not a problem. If he needs the shares, the broker will get them back one way or the other. Let us assume that the broker borrows the shares from another investor who we call Lender. Assume that the short seller sells the stock to another investor, whom we shall call Another.

Now let the stock pay a dividend. Lender expects to receive the dividend because as far as Lender is concerned, he has shares on deposit at broker Broker. But Another also expects to receive the dividends, because Another purchased the shares in good faith

from Short. Another is unaware that the shares have been borrowed, and it would not difference anyway. So investors Another and Lender both have a claim on the dividends. This conflict is resolved by having Short pay the dividends. The company dividend check goes to Another. Short pays the dividend to his broker who gives the money to Lender, who is never aware that anything out of the ordinary happened. Indeed nothing out of the ordinary did happen. This practice occurs all of the time.

In the above formula for the return when there are dividends, we had

$$r_t = \frac{S_t + D_t}{S_0} - 1.$$

Consider an investor who borrows a \$100 stock. The stock price goes to \$95 and there is a \$2 dividend. What is the rate of return for the short seller?

$$r_t = \frac{\$95 + \$2}{\$100} - 1 = -3\%.$$

In effect, this investor sold the stock for \$100 and bought it for \$97, consisting of \$95 to buy the stock and \$2 to make up of the dividend. Note how the general formula for rate of return still works.

Margin Trading and Short Selling in Practice

Margin trading and short selling in practice are subject to certain rules that we frequently ignore in trying to understand the basic economic ideas behind these concepts. We will take a look at these practical issues here.

In margin trading there is a limit on the amount of money that the investor can borrow. We need to first define two notions of margin. Recall that the margin is the percentage of the investor's portfolio represented by his own equity. The initial margin is this percentage on any day on which a trade is done. On any other day, the margin is referred to as the maintenance margin. In the United States initial margin requirements are regulated by the Federal Reserve.⁵ Maintenance margin requirements are regulated by the exchange on which the stock is listed. Thus, the NYSE Euronext, NASDAQ OMX,

⁵The Federal Reserve is not ordinarily a regulator of the stock market. Its margin regulatory authority is granted through an interpretation of the Federal Reserve Act of 1913, which gives the Federal Reserve authority to regulate any form of credit.

and various regional exchanges regulate maintenance margins. In addition, individual brokers can add to the maintenance margin requirement. The Federal Reserve's initial margin requirement is 50%, meaning that the initial margin must be at least 50%. Thus, an investor cannot borrow more than 50% of the purchase price of an asset. The maintenance margin requirement is typically 25% to 30%. On the day on which the investor buys a stock, the initial margin requirement applies. Any day thereafter, the maintenance margin requirement applies, though on any day on which a trade is done, the initial margin requirement again applies.

The margin is computed using the following formula:

$$m = \frac{S - B}{S},$$

where m is the margin, S is the value of the stock, and B is the amount borrowed.⁶ Remember that this figure must be at least 50% on any day of a transaction and at least 25% or 30%, depending on the maintenance margin requirement of the exchange and broker, on any day on which a trade is not done. Margin is computed on a portfolio basis, so you can be under-margined on one stock and over-margined on another as long as the overall margin meets the requirements.

As an example, suppose you borrowed \$40 and combined that with \$60 of your own money to buy a \$100 stock. Your margin is 60% so it exceeds the 50% minimum. Let the maintenance margin requirement be 30%. Now let the stock increase the next day to \$105. Your margin is

$$m = \frac{\$105 - \$40}{\$105} = 61.91\%.$$

Thus, you are well above the maintenance margin requirement. Now suppose the stock falls to \$70. Your margin is

$$m = \frac{\$70 - \$40}{\$70} = 42.86\%.$$

You are still well above the maintenance margin requirement. Suppose the stock keeps falling and hits \$50. Your margin is now

⁶Technically B would include any interest owed on the margin loan.

$$m = \frac{\$50 - \$40}{\$50} = 20\%.$$

You are now below the 30% maintenance margin requirement. You will get a margin call from your broker and will need to deposit some additional funds or the broker will need to sell your stock. The amount of funds you would need to deposit is easily found. Simply set m to 0.30 and insert an unknown value D into the numerator. Then solve for D :

$$0.30 = \frac{S - B + D}{S}$$

$$D = -0.70S + B$$

In this example, we have

$$D = -0.70(\$50) + \$40 = \$5.$$

Thus, if you deposited \$5, you would have assets of \$55 and debt of \$40 for equity of \$15, which is 30% of the value of the stock of \$50. If you do not wish to deposit additional funds, the broker will sell your stock. You would be liable if the broker is unable to sell the stock for at least the amount of the debt.

Another useful calculation is to determine how far the stock would have to fall to get a margin call. Let S^* be the price at which you would have 30% margin. Substitute into the above formula

$$m = \frac{S^* - B}{S^*}$$

$$S^* = \frac{B}{1 - m}.$$

In our example, that would be

$$S^* = \frac{\$40}{1 - 0.30} = \$57.14.$$

Thus, at a stock price of \$57.14, the investor would have $\$57.14 - \$40 = \$17.14$ of equity, which is 30% of the stock price of \$57.14.

Now let us look at short selling in practice. Let us say that an investor borrows \$100 of stock and sells short. He receives the \$100, which we will assume is deposited into an account. The investor cannot withdraw the \$100. It serves as a form of collateral to ensure that the investor can repurchase the stock. In addition, the investor must meet

the initial and maintenance margin requirements. Thus, to meet the initial margin requirements the investor will need to deposit another \$50, which he must come up with from another source. Thus, the investor must deposit \$150 to meet the 50% margin requirement. This is obtained as follows. Let P be the proceeds of the short sale and I be the initial margin requirement. Then,

$$m = \frac{P + I - S}{S}.$$

Here,

$$m = \frac{\$100 + \$50 - \$100}{\$100} = 0.50.$$

That is, the investor's margin account contains \$150. From this, \$100 is the value of the stock and therefore represents the value of the investor's current debt, which is his obligation to buy back the stock. The equity is \$50 or 50% of the value of the stock.

Now let the stock go up to \$110. Then the margin is

$$m = \frac{\$150 - \$110}{\$110} = 36.36\%.$$

Thus, the account is above the maintenance margin requirement.⁷ But now let the stock go to \$130. Then the margin is

$$m = \frac{\$150 - \$130}{\$130} = 15.39\%.$$

This amount is below the maintenance margin requirement. Thus, the investor will have to deposit additional funds to get the equity up to 30% of the stock price of \$130 or \$39. Since the stock is worth \$130, the margin account balance needs to get up to \$169. Since the account has \$150 in it, the investor must deposit \$19, which is found as follows. Let D be the additional amount deposited:

$$m = \frac{P + I - S + D}{S} = 0.30.$$

Solving for D gives

⁷We continue with the assumption of a 30% maintenance margin requirement.

$$\begin{aligned}
 D &= S(1 + m) - I - P \\
 &= \$130(1.30) - \$100 - \$50 \\
 &= \$19
 \end{aligned}$$

The price that would trigger a margin call is found using S^* as the critical price in the formula above:

$$\begin{aligned}
 m &= \frac{P + I - S^*}{S^*} \\
 S^* &= \frac{P + I}{1 + m} \\
 &= \frac{\$100 + \$50}{1.30} = \$115.38
 \end{aligned}$$

That is, if the stock is at \$115.38, the margin would be $\$150 - \$115.38 = \$34.62$, which is 30% of \$115.62.

In the stock market, the number of shares sold short is called the short interest. Some investors consider this figure an indicator of potential buying power, but that is a dubious claim. As long as an investor meets the margin requirements, it can maintain a short position indefinitely. Thus, some shares might never be repurchased, and certainly all are not likely to be repurchased at the same time. Thus, the repurchases, if they occur, can be spread out over an infinite period of time. Thus, short interest is a doubtful indicator that the market will rise. It might, however, be a general indicator of the degree or pessimism of investors.

Margin deposits can be met by posting other securities. Hence, an investor could post a U. S. Treasury bill and continue to earn interest. He could also post other stocks or bonds. Effectively that is what is done when margin is computed on the portfolio level. Some securities can meet the margin requirements for others.